Advanced Automatic Control MDP 444

“ If you have a smart project, you can say "I'm an engineer" ”

Lecture 4

Staff boarder

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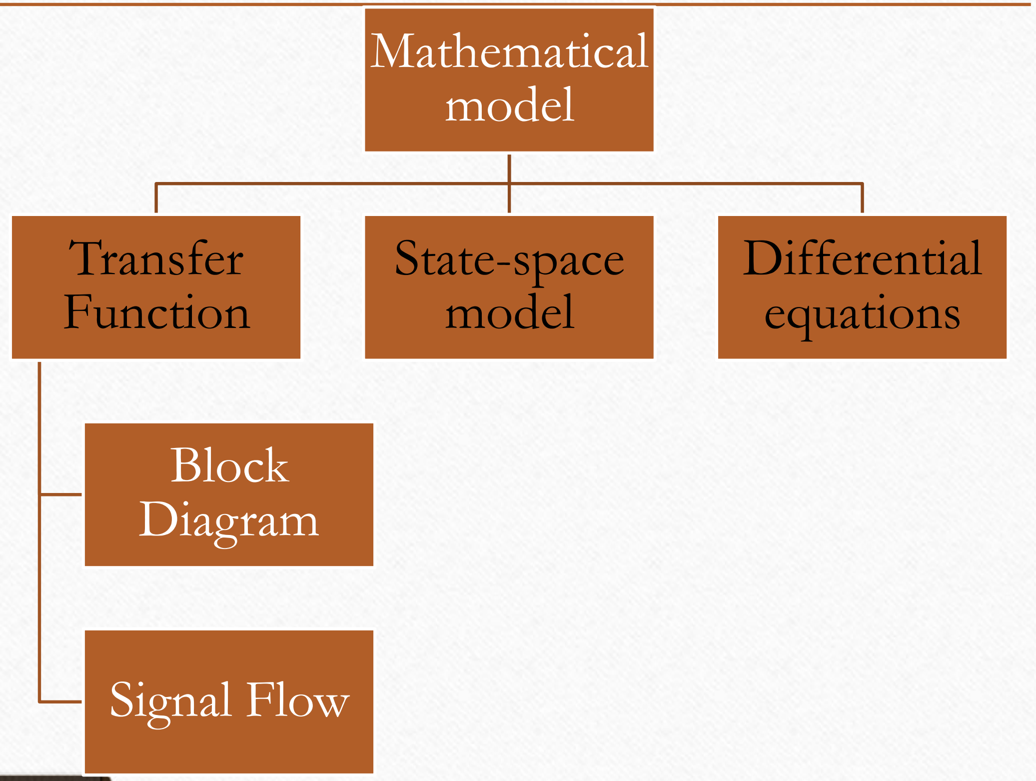
Advanced Automatic Control
MDP 444

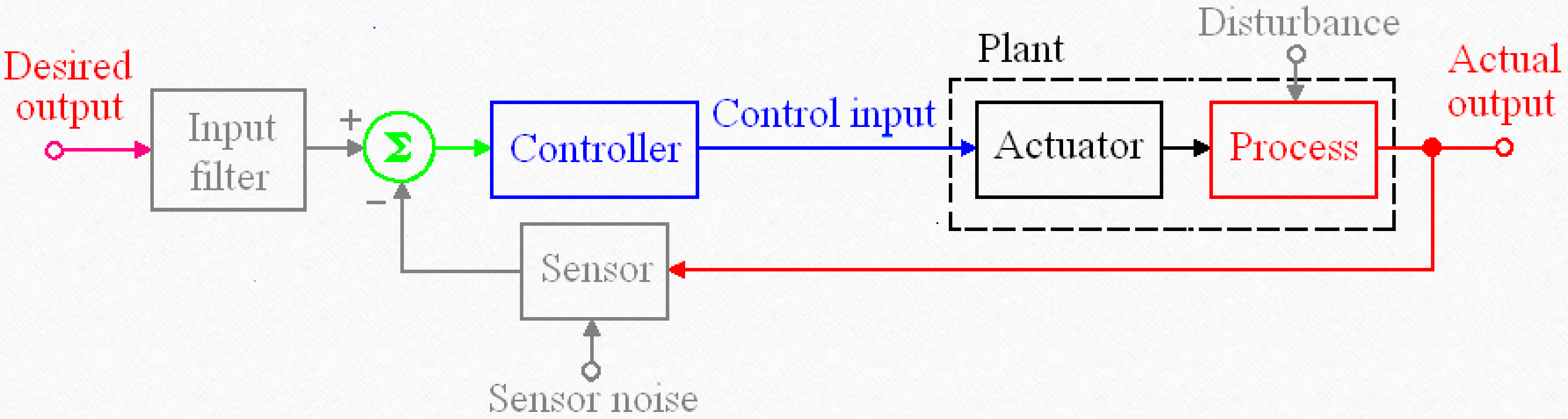
- Lecture aims:
  - Understand the Block reduction techniques
  - Identify the transfer function
  - Be aware by modling multiple technique

Mathematical Modeling

- Transfer Function



Component Block Diagram



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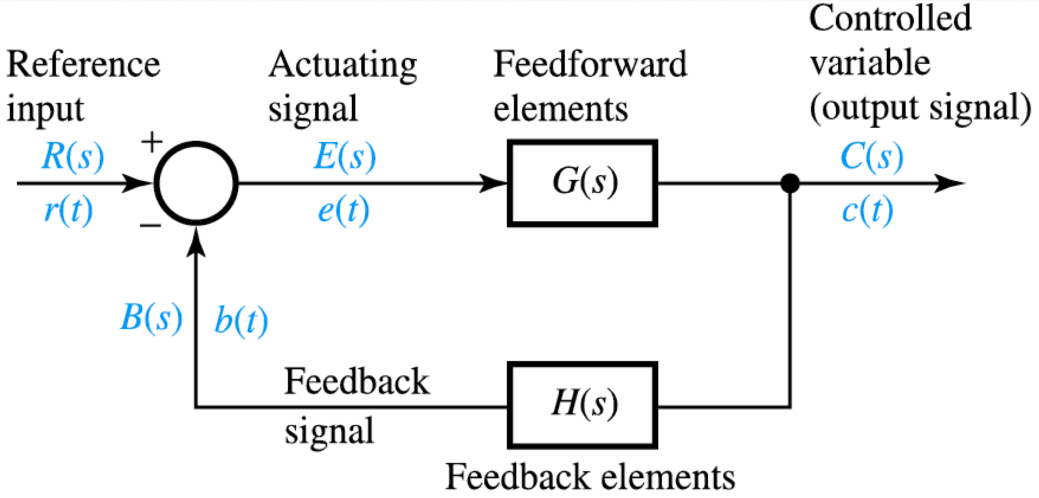
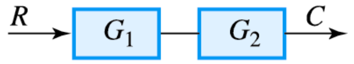
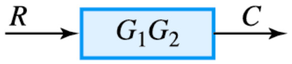
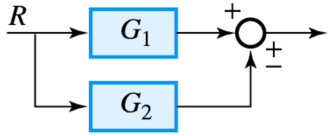
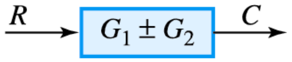
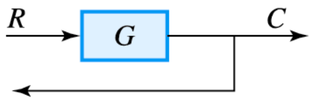
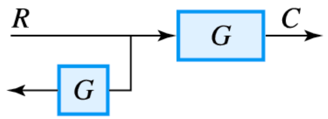
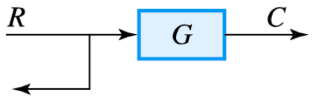
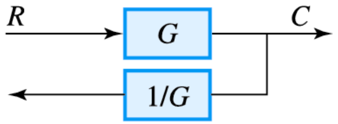
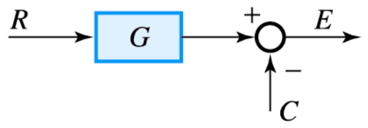
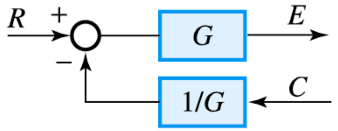
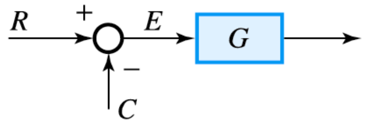
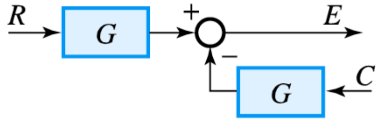
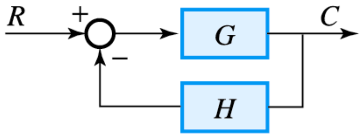
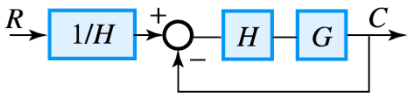
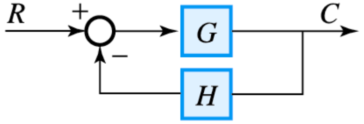
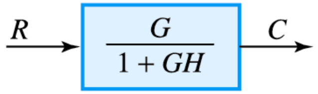


$R(s)$	Reference input
$C(s)$	Output signal (controlled variable)
$B(s)$	Feedback signal = $H(s)C(s)$
$E(s)$	Actuating signal (error) = $[R(s) - B(s)]$
$G(s)$	Forward path transfer function or open-loop transfer function = $C(s)/E(s)$
$M(s)$	Closed-loop transfer function = $C(s)/R(s) = G(s)/[1 + G(s)H(s)]$
$H(s)$	Feedback path transfer function
$G(s)H(s)$	Loop gain

$$\frac{E(s)}{R(s)} = \text{Error-response transfer function} \quad \frac{1}{1 + G(s)H(s)}$$

TABLE 3.4.1 Some of the Block Diagram Reduction Manipulations

Original Block Diagram	Manipulation	Modified Block Diagram
	Cascaded elements	
	Addition or subtraction (eliminating auxiliary forward path)	
	Shifting of pickoff point ahead of block	
	Shifting of pickoff point behind block	
	Shifting summing point ahead of block	
	Shifting summing point behind block	
	Removing H from feedback path	
	Eliminating feedback path	

Signal Flow

A **signal-flow graph** is a diagram consisting of nodes that are connected by several directed branches and is a graphical representation of a set of linear relations.

The basic element of a signal-flow graph is a unidirectional path segment called a **branch**

A **loop** is a closed path that originates and terminates on the same node. Two loops are said to be **nontouching** if they do not have a common node

$$T_{ij} = \frac{\sum_k P_{ijk} \Delta_{ijk}}{\Delta},$$

P_{ijk} = gain of kth path from variable x_i to variable x_j,

Δ = determinant of the graph,

Δ_{ijk} = cofactor of the path P_{ijk},

$$\Delta = 1 - \sum_{n=1}^{N} L_n + \sum_{\substack{n,\,m \\ \text{nontouching}}} L_n L_m - \sum_{\substack{n,\,m,\,p \\ \text{nontouching}}} L_n L_m L_p + \cdots$$

$\Delta = 1 -$ (sum of all different loop gains)

$+$ (sum of the gain products of all combinations of two nontouching loops)

$-$ (sum of the gain products of all combinations of three nontouching loops)

$+ \cdots$.

The cofactor Δ_{ijk} is the determinant with the loops touching the kth path removed.

Signal Flow

The paths connecting the input *R(s)* and output *Y(s)* are

$$P_1 = G_1G_2G_3G_4 \text{ (path 1)} \quad \text{and} \quad P_2 = G_5G_6G_7G_8 \text{ (path 2)}$$

There are four self-loops:

$$L_1 = G_2H_2, \quad L_2 = H_3G_3, \quad L_3 = G_6H_6, \quad \text{and} \quad L_4 = G_7H_7$$

Loops *L1* and L2 do not touch L3 and L4. Therefore, the determinant is

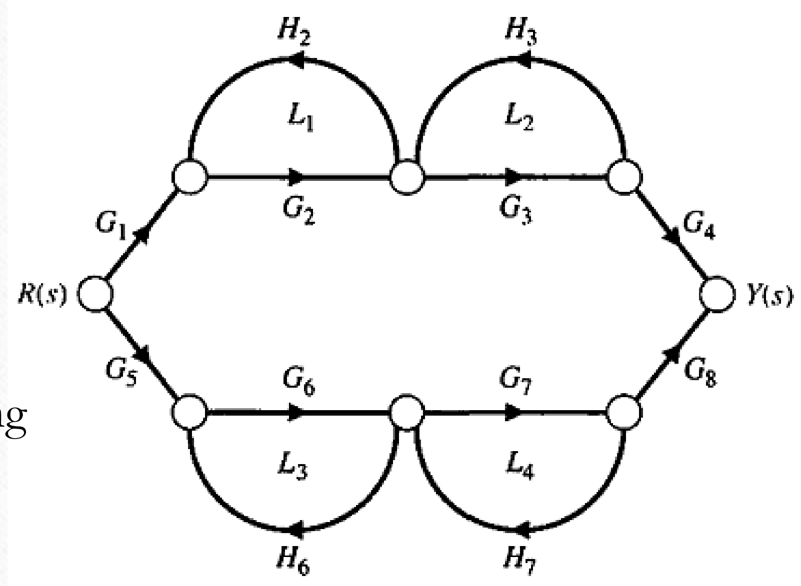
$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4)$$

The cofactor of the determinant along path 1 is evaluated by removing the loops that touch path 1 from Δ. $\Delta_1 = 1 - (L_3 + L_4)$

Similarly, the cofactor for path 2 is $\Delta_2 = 1 - (L_1 + L_2)$

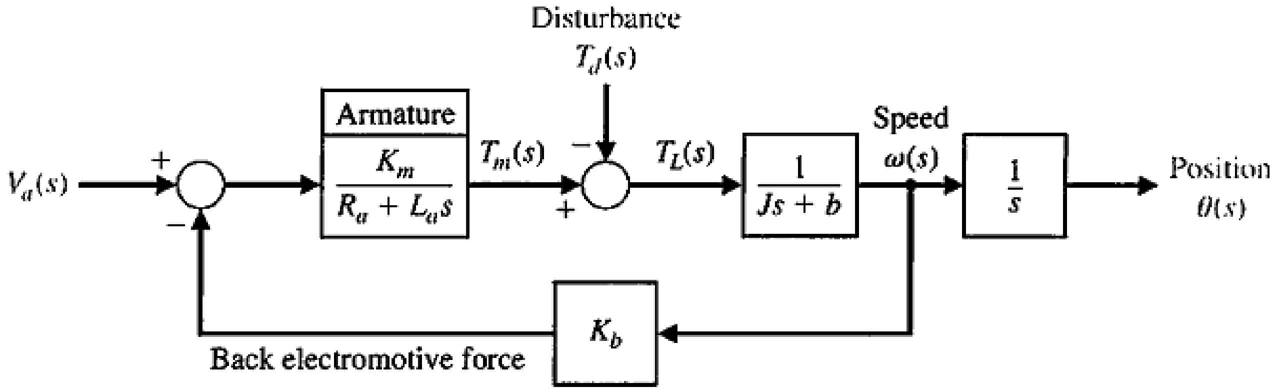
Therefore, the transfer function of the system is

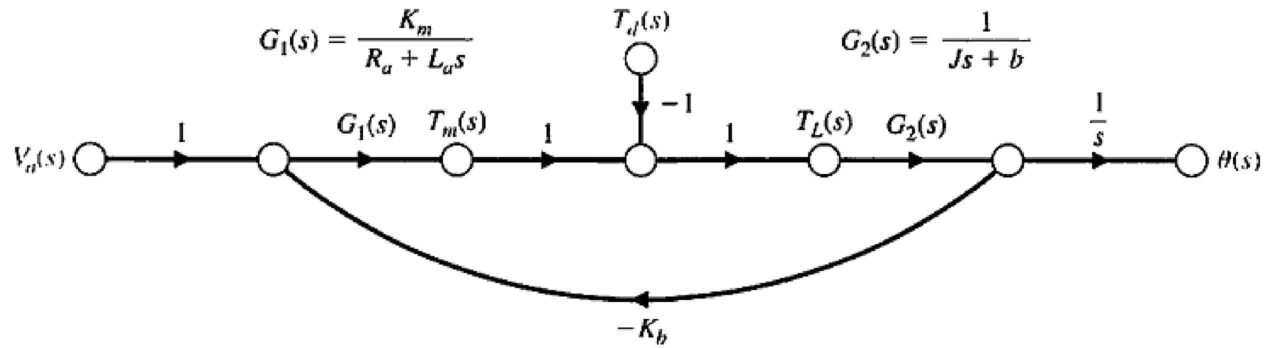
$$\frac{Y(s)}{R(s)} = T(s) = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3G_4(1 - L_3 - L_4) + G_5G_6G_7G_8(1 - L_1 - L_2)}{1 - L_1 - L_2 - L_3 - L_4 + L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4}$$



Signal Flow

The armature-controlled DC motor

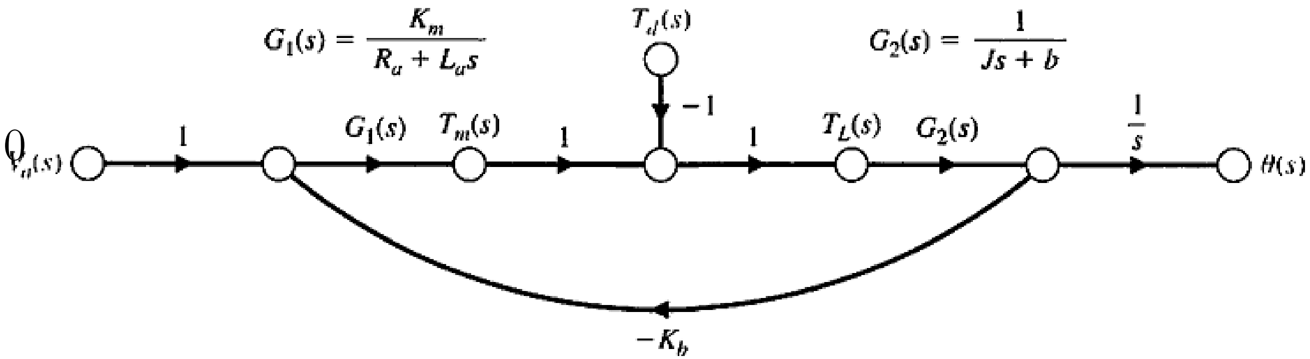




Signal Flow

The armature-controlled DC motor

Using Mason's signal-flow gain formula, transfer function for *θ(s)/Va(s)* with *Td(s)* = 0



The forward path is *P1(s),* which touches the one loop, *L1(s),* where

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State Space Equations

- **State equations** is a description which relates the following four elements: input, system, state variables, and output

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$$

Matrix A has dimensions nxn and it is called the **system** matrix, having the general form

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

Matrix B has dimensions nxm and it is called the **input** matrix, having the general form

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State Space

- **The general form of a dynamic system**

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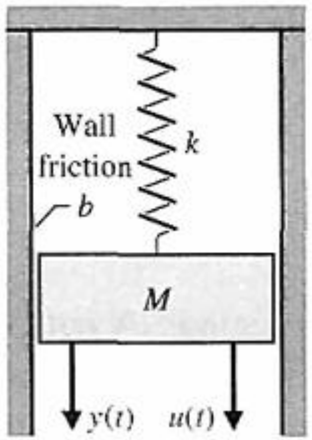

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- **State space matrix**

$$\frac{dx_1}{dt} = x_2$$

- State space matrix

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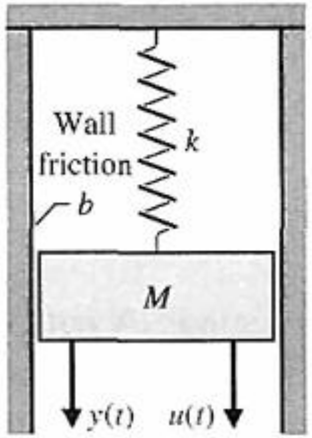

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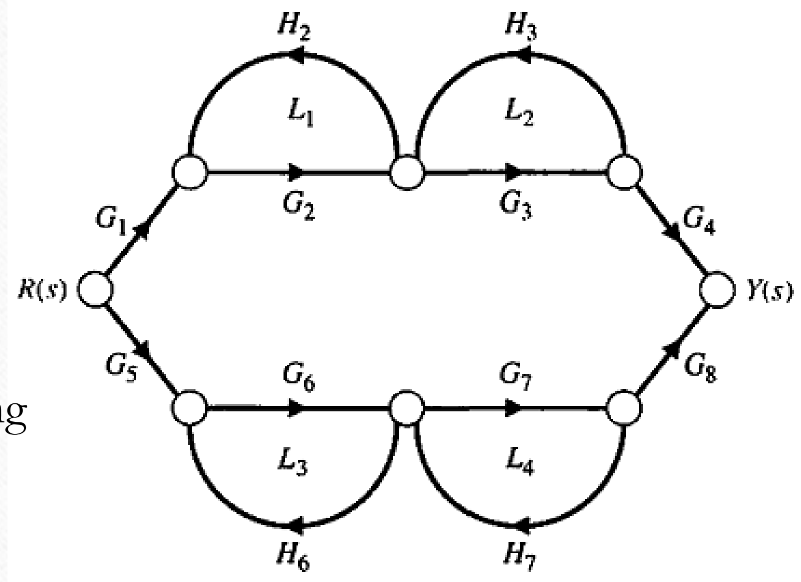

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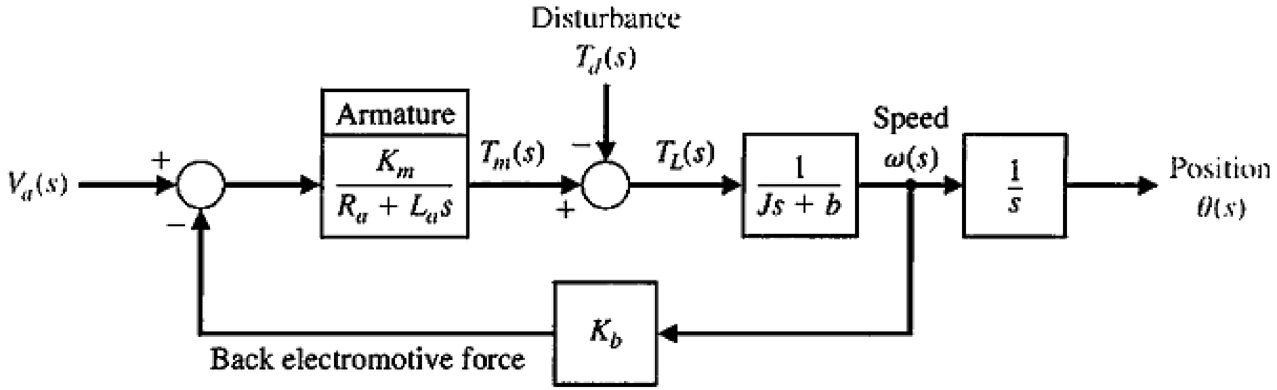

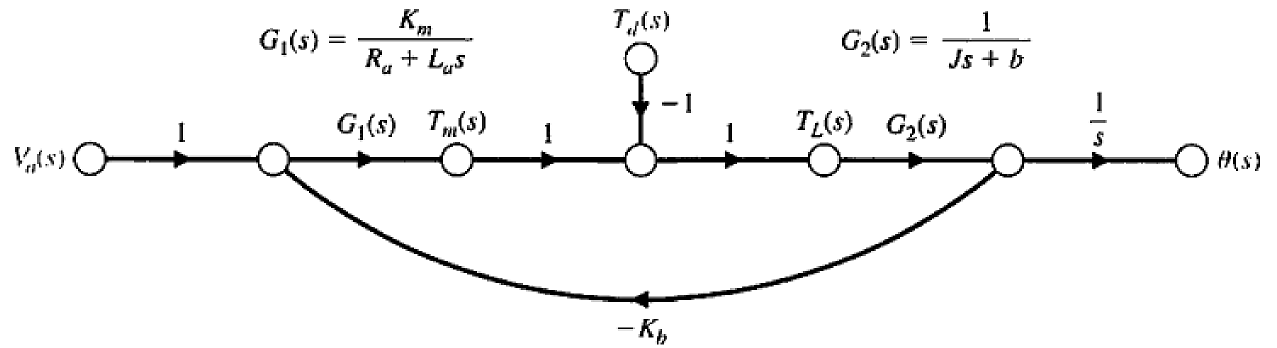

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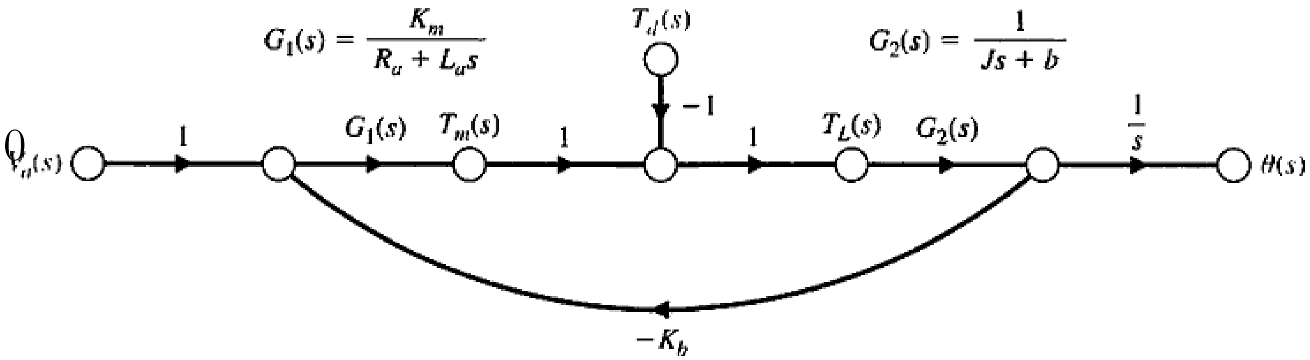

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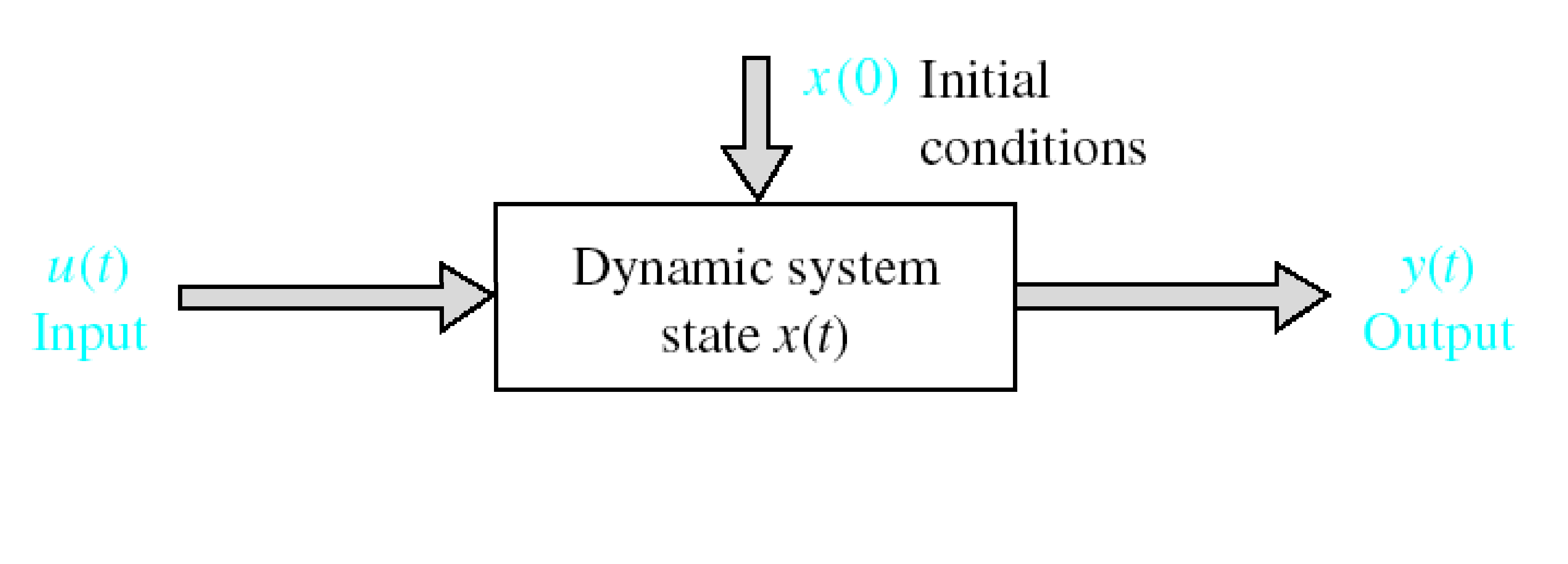
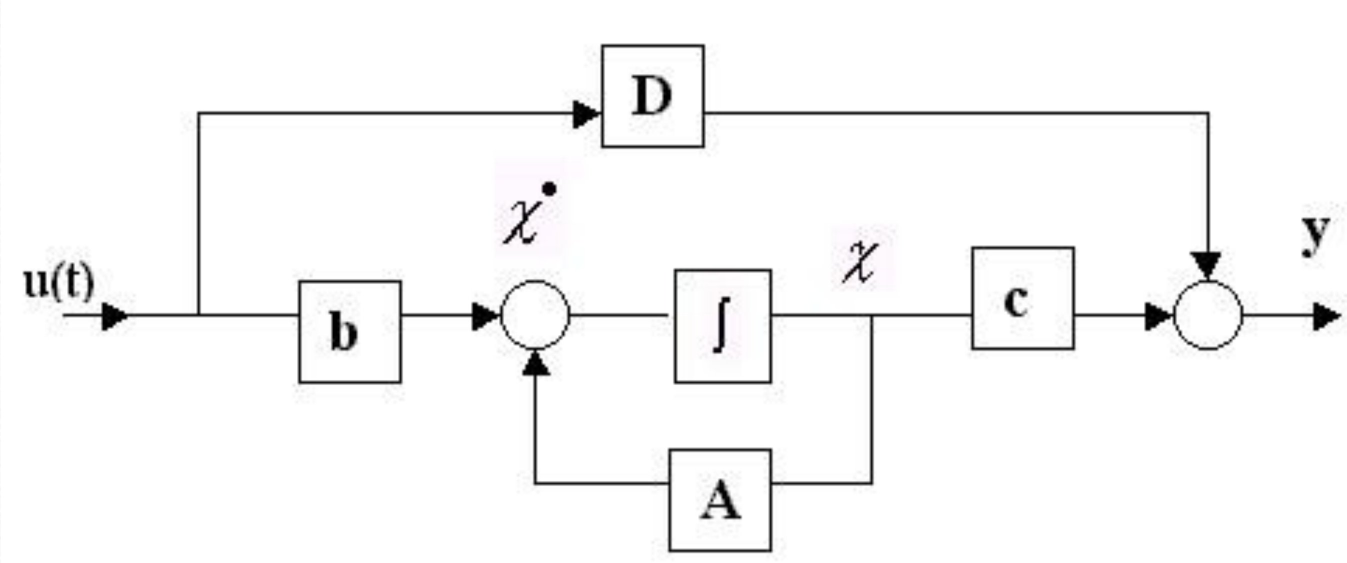
The general form of a dynamic system



State Space Equations

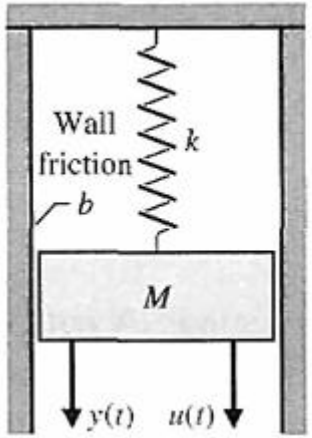
$$SISO \Rightarrow \dot{X} = Ax(t) + Bu(t)$$
$$Y = Cx(t) + Du(t)$$



State Space Representation

- **The general form of a dynamic system**

The concept of a set of state variables that represent a dynamic system can be illustrated in terms of the spring-mass-damper system. A set of state variables sufficient to describe this system includes the position and the velocity of the mass.



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State Space Representation

- **State space matrix**

$$\frac{dx_2}{dt} = \frac{-b}{M}x_2 - \frac{k}{M}x_1 + \frac{1}{M}u$$

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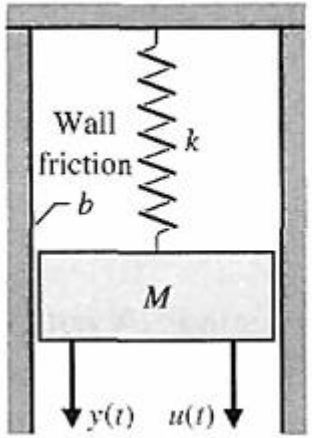

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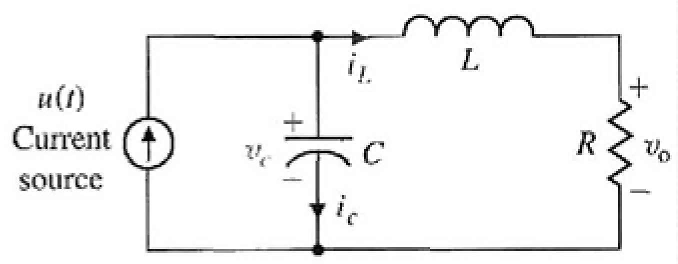
State Space Representation

- **RLC circuit example**
- The state of this system can be described by a set of state variables *(x1, x2)*, where *x1* is the capacitor voltage *vo(t)* and *x2* is the inductor current $i_L\{t)$.
- Utilizing Kirchhoff's current law at the junction



$$i_c = C\frac{dv_c}{dt} = +u(t) - i_L$$

OR

Kirchhoff's voltage law for the right-hand loop provides the equation describing the rate of change of inductor current as

$$L\frac{di_L}{dt} = -Ri_L + v_c$$

The output of this system is represented

$$v_o = Ri_L(t)$$

State Space Representation

RLC circuit example

- rewrite Equations as a set of two first-order differential equations in terms of the state variables *x1* and *x2* as follows:

$$\frac{dx_1}{dt} = -\frac{1}{C}x_2 + \frac{1}{C}u(t) \qquad \frac{dx_2}{dt} = +\frac{1}{L}x_1 - \frac{R}{L}x_2$$

- The output signal is then

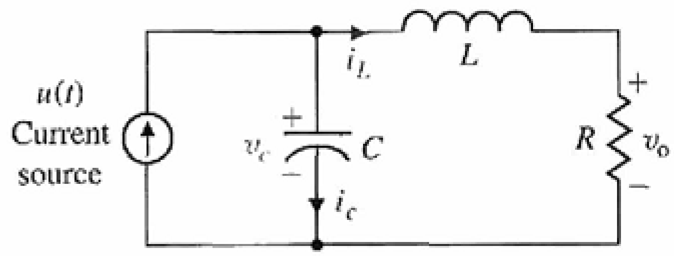
$$y_1(t) = v_o(t) = Rx_2$$

- obtain the state variable differential equation for the *RLC*

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & \frac{-1}{C} \\ \frac{1}{L} & \frac{-R}{L} \end{bmatrix}\mathbf{x} + \begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix}u(t)$$

- and the output as

$$y = [0 \quad R]\mathbf{x}$$



$$i_c = C\frac{dv_c}{dt} = +u(t) - i_L$$

$$L\frac{di_L}{dt} = -Ri_L + v_c$$

$$v_o = Ri_L(t)$$

TRANSFER FUNCTION FROM THE STATE EQUATION

- Obtain a transfer function *G(s),* Given the state variable equations. Recalling Equations :where v is the single output and *u* is the single input.

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x} + \mathbf{D}u$$

  The Laplace transforms of Equations

$$s\mathbf{X}(s) = \mathbf{A}\mathbf{X}(s) + \mathbf{B}U(s)$$
$$Y(s) = \mathbf{C}\mathbf{X}(s) + \mathbf{D}U(s)$$

  where B is an *n* x 1 matrix, since *u* is a single input, we obtain

$$(s\mathbf{I} - \mathbf{A})\mathbf{X}(s) = \mathbf{B}U(s)$$
$$\mathbf{X}(s) = \Phi(s)\mathbf{B}U(s)$$

- we obtain state transition Matrix

$$[s\mathbf{I} - \mathbf{A}]^{-1} = \Phi(s)$$

TRANSFER FUNCTION FROM THE STATE EQUATION

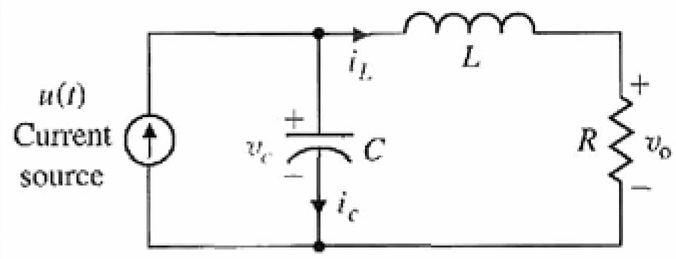
- Transfer function *G(s)*: *G(s)* = *Y(s)*/*U(s)* is

$$G(s) = \mathbf{C}\Phi(s)\mathbf{B} + \mathbf{D}$$

- Let us determine the transfer function *G(s)* = *Y(s)*/*U(s)* for the *RLC* circuit, described by the differential equations

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & \frac{-1}{C} \\ \frac{1}{L} & \frac{-R}{L} \end{bmatrix}\mathbf{x} + \begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix} u$$

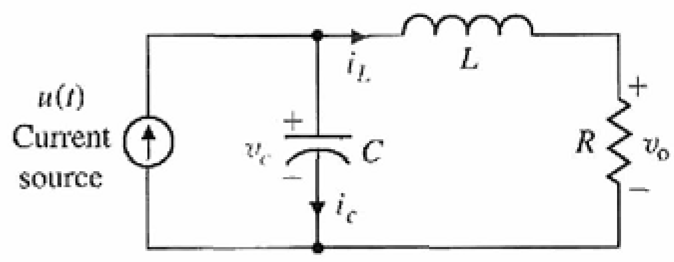
$$y = [0 \quad R]\mathbf{x}.$$



TRANSFER FUNCTION FROM THE STATE EQUATION

- Then we have

$$[s\mathbf{I} - \mathbf{A}] = \begin{bmatrix} s & \frac{1}{C} \\ \frac{-1}{L} & s + \frac{R}{L} \end{bmatrix}$$



- Therefore, we obtain

$$\Phi(s) = [s\mathbf{I} - \mathbf{A}]^{-1} = \frac{1}{\Delta(s)} \begin{bmatrix} \left(s + \frac{R}{L}\right) & \frac{-1}{C} \\ \frac{1}{L} & s \end{bmatrix}$$

$$\Delta(s) = s^2 + \frac{R}{L}s + \frac{1}{LC}$$

- Then the transfer function is

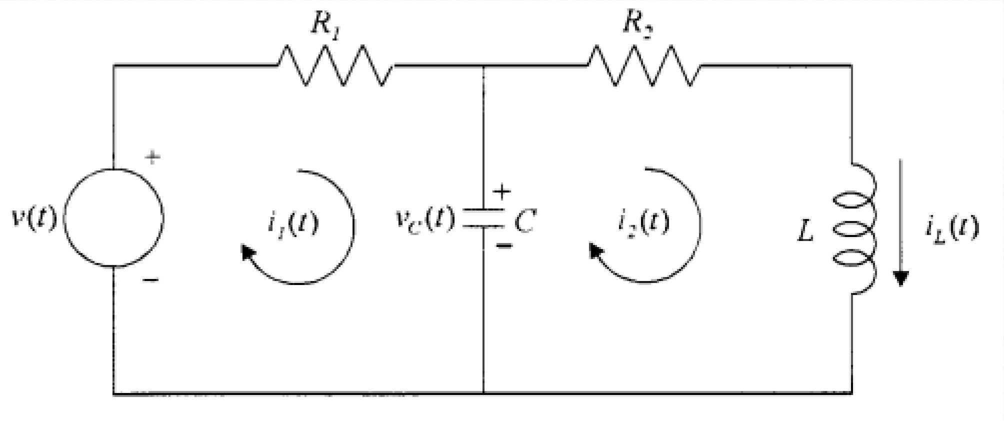
$$G(s) = [0 \quad R] \begin{bmatrix} \frac{s + \frac{R}{L}}{\Delta(s)} & \frac{-1}{C\Delta(s)} \\ \frac{1}{L\Delta(s)} & \frac{s}{\Delta(s)} \end{bmatrix} \begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix} = \frac{R/(LC)}{\Delta(s)} = \frac{R/(LC)}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

State Space representation

- Transfer from time domain to frequency domain:

$$R_1 i_1(t) + \frac{1}{C}\int_0^t i_1(t)\,\mathrm{d}t - \frac{1}{C}\int_0^t i_2(t)\,\mathrm{d}t = v(t)$$

$$\left[R_1 + \frac{1}{Cs}\right] I_1(s) - \frac{1}{Cs} I_2(s) = V(s)$$



$$-\frac{1}{C}\int_0^t i_1(t)\,\mathrm{d}t + R_2 i_2(t) + L\frac{\mathrm{d}i_2}{\mathrm{d}t} + \frac{1}{C}\int_0^t i_2(t)\,\mathrm{d}t = 0$$

$$-\frac{1}{Cs} I_1(s) + \left[R_2 + Ls + \frac{1}{Cs}\right] I_2(s) = 0$$

- Transfer function

$$\frac{I_2(s)}{V(s)} = \frac{Cs}{(R_1Cs+1)(LCs^2+R_2Cs+1)-1} = \frac{1}{R_1LCs^2 + (R_1R_2C+L)s + R_1 + R_2}$$

State Space representation

$$\begin{cases} e(t) - R_1 i_1(t) - L_1 \dfrac{di_1}{dt} - V_C(t) = \phi \\ V_C(t) - L_2 \dfrac{di_2}{dt} - R_2 i_2 = \phi \\ i_c = i_1 - i_2 = C \dfrac{dv_c}{dt} \end{cases}$$

$$x = (i_1 \quad i_2 \quad v_c)^T$$

$$X^{\bullet} = \begin{pmatrix} \dfrac{-R_1}{L_1} & 0 & \dfrac{-1}{L_1} \\ 0 & \dfrac{-R_2}{L_2} & \dfrac{1}{L_2} \\ \dfrac{1}{C} & \dfrac{-1}{C} & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} \dfrac{1}{L_1} \\ 0 \\ 0 \end{pmatrix} e(t)$$

$$y(t) = (0 \quad R_2 \quad 0) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

State Space representation

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Laplace

$$\begin{cases} S\,x(s) - x(0) = Ax(s) + Bu(s) \\ Y(s) = C\,x(s) + Du(s) \end{cases}$$

$$\left. \begin{aligned} X(S) &= (SI - A)^{-1} Bu(s) \\ y(s) &= (C(SI - A)^{-1} B + D)u(s) \end{aligned} \right\}$$

Laplace

$$G(S) = C(SI - A)^{-1} B + D$$

$$G(S) = \frac{Q(S)}{|SI - A|}$$

State Space representation

$$\begin{cases}\dot{x}_1 = -5x_1 - x_2 + 2u \\ \dot{x}_2 = 3x_1 - x_2 + 5u\end{cases} \Rightarrow \dot{x} = \begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix} x + \begin{pmatrix} 2 \\ 5 \end{pmatrix} u$$

$$y = x_1 + 2x_2$$

$$y = (1 \quad 2)\, x$$

$$(SI - A) = \begin{pmatrix} 2+5 & 1 \\ -3 & S+1 \end{pmatrix}$$

$$(SI - A)^{-1} = \frac{1}{\underbrace{(S+5)(S+1)+3}_{\Delta=(S+2)(S+4)}} \begin{pmatrix} S+1 & -1 \\ 3 & S+5 \end{pmatrix}$$

$$G(S) = [1 \quad 2] \frac{1}{\Delta} \begin{bmatrix} S+1 & -1 \\ 3 & S+5 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$G(s) = \frac{12S + 59}{(S+2)(S+4)}$$

Model Examples

- Pulse Width Modulation (PWM)